Photogrammetry: An Available Surface Characterization Tool for Solar Concentrators, Part II: Assessment of Surfaces

In a previous paper, the results of photogrammetric measurements of a number of paraboloidal reflecting surfaces were presented. These results showed that photogrammetry can provide three-dimensional surface characterizations of such solar concentrators. The present paper describes the assessment of the quality of these surfaces as a derivation of the photogrammetrically produced surface coordinates. Statistical analysis of the z-coordinate distribution of errors indicates that these generally conform to a univariate Gaussian distribution, while the numerical assessment of the surface normal vectors on these surfaces indicates that the surface normal deviations appear to follow an approximately bivariate Gaussian distribution. Ray tracing of the measured surfaces to predict the expected flux distribution at the focal point of the 400 m² dish show a close correlation with the videographically measured flux distribution at the focal point of the dish.

Introduction

Photogrammetry utilizes the principle of stereoscopic reconstruction of a three-dimensional surface through the use of photographs or images taken of the surface from two or more viewing positions. The three-dimensional coordinates of specific targets placed on the surface are determined through a combination of observations made on common targets in different photographs, and the application of photogrammetric processing software to provide a least-squares adjustment of resultant data arrays, which in turn produces the estimated target coordinates and associated precisions. A previous paper (Shortis and Johnston, 1995) described the application of photogrammetry to characterize the surfaces of four separate sets of reflecting surfaces.

These measured surfaces included the surface of a 5-m diameter, circular aperture paraboloidal dish solar concentrator, a 30-cm square, curved mirror tile, a 4.2-m triangular mirror panel that supports approximately 70 of these tiles, and finally the surface of the ANU 400 m² dish, that is covered with 54 of these 4.2-m mirror panels. In addition to these, a further seven 60-cm mirror tiles and an equal number of their associated triangular mirror panels have been measured using digital photogrammetry. This technique has provided a higher coordinate accuracy, data point density, and much reduced processing time compared to the film based technique described previously.

The seven additional mirror tiles and panels were chosen such that one tile from each panel was measured. Ideally, a total of nine panels and tiles should have been measured to accommodate all the panel types used on the dish, but due to difficulties photographing the ninth panel, it was omitted, and the fifth panel was duplicated and used at the coordinates applicable to the ninth panel. Figure 1 shows the overall layout on the 400 m² dish surface, also illustrating the positions of the measured panels and tiles.

Further analysis is required to assess the quality of the measured surfaces. Two measures of ideality have been used to compare the surfaces in the present study with the expected paraboloidal shape. The first looks at the deviation of the z-coordinates of the measured surfaces from the z-coordinates of a corresponding ideal paraboloid. This allows a visual inspection to be made of the surface perturbations as they exist in three-dimensional space. The second method numerically calculates the surface normal vectors of a measured surface, and then assesses the angular deviations (surface normal error) of these vectors from the normal vectors expected for an ideal paraboloid. This latter metric appears to be the more widely utilized measure of the “quality” of a solar concentrator reflecting surface, and is often referred to as “surface slope error” or simply “slope error” (Romero, 1994), (Grossman et al., 1992), (Taneja et al., 1992), (Krasilovskii et al., 1978). There will be some correlation between figures of merit based on the two quality measures. However, direct comparison is qualitative at best, as surface slope error is dependant on both the magnitude and relative spacing (spatial frequency) of the z-coordinate deviations, and the effect of both of these quantities on the surface slope error is difficult to quantify without doing a full surface normal analysis.

An approximate cross-check on the validity of the calculated normal vectors and slope errors can be performed by using a ray trace algorithm to simulate the focal flux that would be produced from the photogrammetrically measured surfaces, and to compare this distribution with the measured distribution for the concentrator. Comparisons with a simulated ideal paraboloidal surface having an assigned level of random surface error (which follows a bivariate Gaussian distribution) can also be made. Depending on the degree of closeness to a bivariate Gaussian distribution of errors in the measured surface, the two flux distributions should show a significant degree of correlation.

Assessment of Surfaces

Z-Coordinate Deviations. All surfaces should ideally fit the equation to a paraboloid, that is,

\[ z = \frac{1}{4f} \left( x^2 + y^2 \right) \]

where \( f \) is the focal length.
Plotting the $z$-coordinates versus $r^2/f$ should yield a straight line with a slope of $1/f$. A least-squares linear fit was performed on such plots for the 5-m dish, the eight triangular mirror panels and the 400 m$^2$ dish, the 30-cm mirror tile, and the seven 60-cm tiles. Following this, the focal lengths were used to construct ideal paraboloidal surfaces with which to compare the deviation of $z$-coordinates of the measured surfaces from the ideal. When plotted as a frequency distribution, the $z$-coordinate deviations appeared to follow an approximately Gaussian distribution of values. The standard deviations of these distributions can be used as a measure of the overall “ripple” in the surfaces. Figures 2 and 3 show the surface deviation plot and the frequency distribution of surface deviations, respectively for the 5-m dish. Similar analyses were performed for the other reflector surfaces. Table 1 summarizes the corresponding results.

**Surface Normal Determinations**

**Development of a Surface Normal Calculation Routine.**

Of primary interest in solar concentrator assessment is a measure of the deviation of the surface normals away from the ideal normal direction (slope error). A numerical surface normal calculation routine, dubbed GRADFITTER, has been developed to assess surface normals for numerically defined surfaces. It utilizes a bicubic B-spline fit to the surface data points using automatic knot placement according to the value specified in a smoothing variable. This variable was adjusted such that closest approximation to local variations was obtained, while avoiding spurious oscillations and instabilities between knots (knots are anchor points placed in the fitting function through which the surface must pass).

GRADFITTER utilizes the EO2DDF and EO2DEF surface fitting algorithms in the NAG$^1$ library of numerical analysis routines. This routine is in turn based on the SURFIT algorithm contained in the FITPACK analysis package developed by Dierckx (1993). The robustness of fit was maintained by minimizing the $z$-coordinate deviations of the interpolated spline surface compared to a bilinear interpolation of the surface. It was found that the deviations from a bilinear fit were large when the smoothing factor was large (indicating that the spline surface was smoothing the data points too severely), and when the smoothing factor was excessively small (indicating that the spline surface was “rippling” excessively between knots). The deviations were at a minimum when the spline surface most closely approximated the local variations in the data. The surface normals were calculated by taking the cross-product of interpolated elemental vectors within the fitted surface in close proximity to the actual data points. The algorithm accommodates both nonrectangular and nonregular data arrays. The assessment of surface slope errors at a particular surface coordinate was performed by calculating the ideal surface (unit) normal (finding the surface gradient of the paraboloid) and then subtracting this from the calculated surface (unit) normals determined using GRADFITTER (i.e., the radial component of the error displacements was used). The length of this difference vector was then used as the measure of the slope error at the coordinate point. This process assumes that the error distribution is circularly symmetric around the ideal normal direction (i.e., tangential error displacements are assumed uniform, and thus ignored). While this assumption is not always necessarily correct, particularly where systematic surface distortions are apparent, calculation of surface slope error is generally performed, and specified, for the radial error component.

**Surface Fitting Accuracy Assessment.**

Determining the accuracy of fit for a general spline curve or surface to unknown data can prove a difficult exercise. Experience with spline fits shows that they can either smooth a surface excessively, or introduce unrealistic fluctuations between data points, depending on the placement of knots. Data point density also affects the accuracy of fit. The data point density chosen for the measured surfaces was initially based on a visual inspection of the mirror tiles to assess the “characteristic ripple distances” that appeared upon them. These ripple distances were characterized by measuring the distances between apparent peaks and troughs that occurred on a range of tiles in both their $x$ and $y$ dimensions. The mirror tile surfaces were smoothly varying functions across both dimensions—cracked mirrors, containing sharp discontinuities, were not used for assessment.) Target point spatial frequency was chosen to ensure at least six data points per peak (or trough) in both dimensions were

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achieved, and it was found that an average of one peak and/or one trough appeared in either dimension on any (60-cm) mirror tile. It was decided to create a 13 x 13 target array on the 60-cm mirror tiles, thus allowing adequate data coverage for the 1 peak/1 trough combinations that could occur.

The accuracy of fit for GRADFITTER was assessed by first constructing an artificial surface composed of a paraboloid with superimposed sinusoidal ripples whose frequency and amplitude could be varied. Average slope error of the bicubic spline fit was calculated for a range of ripple amplitudes and frequencies. Figure 4 shows the resulting plot of slope error versus ripple frequency for a family of ripple amplitudes. It can be seen that uncertainties of approximately 0.2 milliradian or less in the surface normal determinations can result if surface ripple amplitudes are below ±2 mm and frequencies below 2 m⁻¹. Further definition of error beyond this becomes difficult, as real surfaces will not conform to any easily defined mathematical form. For the present, assessment of the deviations of the measured tile surfaces from an ideal paraboloidal shape indicated that the worst case rippling showed deviations of ±1.5 mm, and approximate low-frequency components of 1.5–2 m⁻¹. It is thus expected that uncertainties in the surface normals calculated by GRADFITTER will be 0.2 milliradian or less.

Figure 5 shows the superposition of a bilinear fit to the measured surface data for a 60-cm tile and the corresponding bicubic spline fit generated using GRADFITTER. (The surfaces shown in Fig. 6 are inverted for clarity—therefore surfaces are concave in reality.) It can be seen that the differences are very minor. Calculations indicate that the confidence in the surface normals, as the data were distributed too sparsely to provide a robust surface normal calculation.) Figure 6 shows an example of the spatial distribution of surface slope errors across the surface of one of the 60-cm mirror tiles. Similar graphical descriptions can be made for other surfaces. While visualisation of the spatial distribution of surface slope error has many practical uses, another useful quantity that can be developed is an estimate of the standard deviation of the distribution of surface normal vector differences. This is the so-called “figure of merit” for a surface. It can be shown (Johnston, 1995a) that if the distribution of surface normal vectors follows a circular bivariate Gaussian probability density function, then the radial distribution of errors (which are measured in the present analysis) can be expected to follow a Rayleigh distribution of the form,

\[
\frac{dP}{dr} = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2},
\]

where \(dP/dr\) is the change in probability as a function of radial
deviation, \( r \), from the center of the distribution and \( \sigma \) is the standard deviation of the distribution. It can also be shown that the peak value (mode) of this distribution defines the standard deviation (\( \sigma \)) of the bivariate Gaussian distribution of surface normals.

Figure 7 shows a statistical frequency distribution plot of the surface normal deviations apparent on the 5-m dish surface. A more refined assessment was applied to the triangular mirror panels by calculating surface normals for their respective mirror tiles, and then duplicating these tiles (with normals) across the surfaces of their appropriate mirror panels.

Finally, the panel and tile arrays were duplicated five times (for each panel type) and the set of six panels was then oriented into their appropriate (photogrammetrically determined) positions on the 400 m\(^2\) dish surface. The deviations between the numerically calculated surface normals and their corresponding ideal directions was performed for the panel and tile combinations, and a statistical frequency distribution analysis performed on these slope errors. Figure 8 shows the distribution of slope errors for an entire dish sector, consisting of nine panels covered with mirror tiles. The figure also shows the least-squares-best-fit Rayleigh distribution (smooth curve) superimposed on the data. Table 2 summarizes the results of the surface normal assessments for the measured surfaces.

**Ray Tracing Based on Photogrammetrically Determined Surface Data**

A computer ray trace algorithm dubbed "COMPREC" (acronym for COMpound RECeiver) was written to model incident flux distributions on receiver surfaces placed in the focal regions of concentrating solar collectors. The program can model receivers having flat, cylindrical, conical, or partial spherical geometries (or any number of combinations of these shapes).

Concentrator surface data is read as an alternating series of coordinate position and surface normal vectors. Surface slope errors which follow a bi-variate circular Gaussian distribution, and are defined by the standard deviation of that distribution, can also be modeled on ideal dish surfaces.

A uniform intensity ("pill-box") sunshape was defined for the algorithm, and the photogrammetrically determined surface definition data for the nine panel types was used as input to COMPREC. The target surface was modelled as a flat square having a side of 1.2 m placed at the 13.1-m focal point of the dish. Predicted fluxes were calculated for the individual panels, for combinations of the six panels of one type (aligned to their measured positions on the 400 m\(^2\) dish surface), and for the entire dish surface covered with panels and mirror tiles. Figures 9(a) and 9(b) show mesh and image plots of the distribution expected from the entire dish surface.

**Comparison of Predicted Flux Results**

Figure 10 shows the superposition of the measured focal flux distribution (heavy line) (Johnston, 1995b) with the photogrammetrically predicted distribution (light line)—the predicted distribution has been scaled to contain the same equivalent power as the measured distribution. Also shown (dotted line) is the simulated distribution for a paraboloidal reflector having 6.5 milliradian of surface slope error, scaled to contain the same integrated power as the measured distribution. Comparing these distributions indicates that the photogrammetric prediction shows some deviations from the measured distribution, particularly a noticeable narrowing near the center of the distributions. Figure 11 shows the percent power within radius calculated for the measured, photogrammetrically predicted and the simulated (6.5 milliradian) flux distributions. Again, the photogrammetrically predicted distribution shows some deviation from the measured distribution.

**Discussion**

The utility of measured surface coordinates in solar collector analysis is apparent in the range of information that can be derived from these measurements. Dish focal lengths, z-coordinate deviations (indicating the degree of ripple in the mirror surfaces), surface normal determinations with associated slope errors and spatial distribution of errors on a dish surface are all available with appropriate processing of the data. The photogrammetric method also allows measurement of nonreflective surfaces, such that assessments can be performed on, say, molds used to fabricate concentrator surfaces, and conformance with design criteria calculated to determine the quality of the concentrator that could be produced from such molds.
Fig. 9(a) Flux distribution predicted from the photogrammetric assessment of the complete ensemble of all tile and panel types constituting the surface of the 400 m² dish

Fig. 9(b) Image of the flux distribution shown in Fig. 9(a)

However, while these derived quantities are valuable for assessment of the quality of a particular collector, the most useful contribution that photogrammetry can offer to solar collector surface measurement is the ability to predict the focal flux distribution that will occur for the collector under test. Review of the flux distributions and power-in-radius plots shown in Figs. 10 and 11 shows that while some deviations between the predicted and measured flux arrays are apparent, overall the predicted distribution shows a close approximation to the measured distribution. Reference to Fig. 10 appears to indicate that the photogrammetrically predicted flux may approximate an equivalent bivariate Gaussian distribution of errors having a standard deviation between 0.6 and 6.5 milliradian. These numbers are supported by the approximating Rayleigh distribution shown in Fig. 8, which predicts a standard deviation of an equivalent Gaussian distribution of 6.9 milliradian, although it can also be seen that the actual frequency distribution data do not conform exactly to the fitted Rayleigh distribution. This indicates that the photogrammetrically measured coordinates, and the subsequent surface normal calculations, are not widely variant from the actual values. However, further assessments of the possible sources of error are required to provide a definitive answer to the observed differences.

Several explanations may be advanced for these differences. These include:

(i) The surface normals calculated by GRADFITTER may have errors greater than 0.2 milliradian in some regions of the mirror tiles that were subsequently duplicated across their respective panels, which were in turn duplicated across the surface of the 400 m² dish.

(ii) The number of mirror tiles and panels that were measured to assess the variations that exist across the dish surface may have been insufficient to represent satisfactorily the full range of features that exist on the surface.

(iii) Changes in the surface topology of the dish may have occurred in the two-year time period between the videographic flux measurements and the photogrammetric measurements of the dish surface. In this case, further focal flux measurements are required to assess the extent of these changes, and to then compare their conformance with the photogrammetrically predicted flux distributions.

An observation can also be made on the photogrammetric process. The majority of the earlier work undertaken in the current presentation was accomplished using film-based photogrammetry. This required capturing images on high resolution photographic film (with subsequent processing of the medium), followed by painstaking visual observation of the targets recorded in the images. This is a tedious, time consuming technique, and has been rapidly surpassed by the capabilities of digital photogrammetry, which uses high resolution digital CCD
cameras to capture the required photogrammetric images, with subsequent processing of images using a digital computer. This technique results in orders of magnitude improvements in processing speed, with an improved accuracy capability factor two to three times better than that achieved with the analog equipment used in the film-based analysis. The additional seven mirror tiles and panels described in the present paper were measured using this advanced photogrammetric method, and, in conjunction with other test objects, have shown that relative coordinate precisions up to 1:70,000 appear to be readily achievable with this technique. The reduced operator processing time means a corresponding reduction in cost in this part of the analysis, while equipment cost reductions in the order of 50 percent over the film-based process are also an attractive feature that accompanies digital photogrammetry.

Conclusions

The results achieved with the measured reflecting surfaces indicate that quantifiable solar concentrator surface coordinates and surface slope errors can be achieved using close-range photogrammetry. This quantification is available with appropriate precisions to allow both solar collector surface quality determinations to be made, and for focal region flux distributions to be estimated based on the measured surface coordinate data.

The accomplishments to date indicate that close range photogrammetry is a viable and available tool to undertake solar collector surface analysis.

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References


